Cross Product

Friday, May 12, 2023 8:50 AM

notation: u, v, w vectors

- MON • $|u|$ is length
- WED • $u \cdot v$ is dot product (#) ⟶ u is orthogonal (perpendicular) to v if $u \cdot v = 0$
- TODAY • $u \times v$ is cross product (new vector)

Shorthand notation:

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

(this is called a determinant)

let $\vec{u} = \langle u_1, u_2, u_3 \rangle$ & $\vec{v} = \langle v_1, v_2, v_3 \rangle$

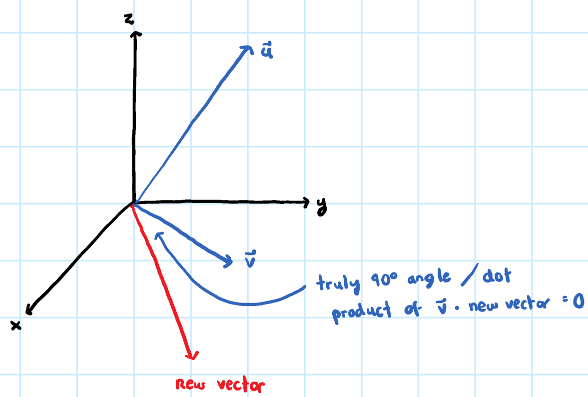
cross product : $\vec{u} \times \vec{v}$ is vector...

$$\vec{u} \times \vec{v} = \langle u_2 v_3 - u_3 v_2,\ -(u_1 v_3 - u_3 v_1),\ u_1 v_2 - u_2 v_1 \rangle$$

ex 1) $\vec{u} = \langle -1, 2, 3 \rangle$, $\vec{v} = \langle 5, 4, 1 \rangle$

then $\vec{u} \times \vec{v} = \langle 2 \cdot 1 - 3 \cdot 4,\ -(-1 \cdot 1 - 3 \cdot 5),\ -1 \cdot 4 - 2 \cdot 5 \rangle$

$$= \boxed{\langle -10, 16, -14 \rangle}$$



(extra ex: $\langle 1, 0, 0 \rangle \times \langle 0, 1, 0 \rangle = \langle 0, 0, 1 \rangle$)

alternatively,

$$\vec{u} \times \vec{v} = \left\langle \begin{vmatrix} u_2 & u_3 \\ v_2 & v_3 \end{vmatrix},\ -\begin{vmatrix} u_1 & u_3 \\ v_1 & v_3 \end{vmatrix},\ \begin{vmatrix} u_1 & u_2 \\ v_1 & v_2 \end{vmatrix} \right\rangle$$

$$= \det \begin{pmatrix} i & j & k \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{pmatrix} = \begin{vmatrix} u_2 & u_3 \\ v_2 & v_3 \end{vmatrix} i - \begin{vmatrix} u_1 & u_3 \\ v_1 & v_3 \end{vmatrix} j + \begin{vmatrix} u_1 & u_2 \\ v_1 & v_2 \end{vmatrix} k$$

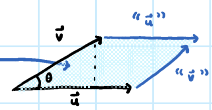
* ignore row & column of letter and cross multiply *

properties of $u \times v$:

1) $\vec{u} \times \vec{v}$ is a vector orthogonal to $\vec{u}$ & $\vec{v}$

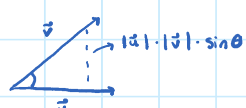
$(\vec{u} \times \vec{v}) \cdot \vec{u} = 0$ $\qquad (u \times v) \cdot \vec{v} = 0$

2) length of $\vec{u} \times \vec{v}$ measures area of parallelogram



cross product - angle formula :

$|\vec{u} \times \vec{v}| = |\vec{u}| \cdot |\vec{v}| \cdot \sin\theta$ ⟶ area of



criterion : $\vec{u}, \vec{v}$ parallel ⟷ $\vec{u} \times \vec{v} = 0$

ex 2) $\vec{u} = \langle -1, 2, 3 \rangle$, $\vec{v} = \langle 5, 4, 1 \rangle$

find area of parallelogram spanned by $\vec{u}$ & $\vec{v}$

solution: it's $|\vec{u} \times \vec{v}|$, since $\vec{u} \times \vec{v} = \langle -10, 16, -14 \rangle$ the length (or area) is ...

$$\sqrt{(-10)^2 + (16)^2 + (-14)^2}$$

$$\boxed{= \sqrt{100 + 16^2 + 14^2}}$$